Université de Bourgogne

Master in Mathematical Physics (Math4Phys)

First year - Program

Language : English
Coordinators: Guido Carlet (guido.carlet@u-bourgogne.fr) and José Luis Jaramillo (jose-luis.jaramillo@u-bourgogne.fr)

Aims

The main aim of the Master is to provide advanced lectures on the mathematical methods of modern theoretical physics in the framework of a mathematical curriculum. Such an offer exists in France only in Dijon as the Mathematical Physics group of the IMB provides a unique environment for a program requiring a double competence in Mathematics and Physics.

Courses of Semester 1 (September - January)

UE1, Differential geometry (7 ECTS)

Differentiable manifolds. Vector fields and flow-box theorem. Differential forms and Stokes' theorem. Tensors and vector bundles. Riemannian manifolds and connections. Geometry of gauge fields.

UE2, Fourier analysis (7 ECTS)

Fourier series for periodic functions of a real variable, Riemann–Lebesgue lemma, Dirichlet's conditions and Parseval's theorem. Convolution of two complex-valued functions defined on $\mathbf{R}^d$. Approximation to the identity. Fourier transform on R=L1 and Fourier inversion theorem. Plancherel theorem and Fourier transform on R=L2. Application to the resolution of some partial differential equations: Schrödinger equations, wave equations and heat equations.

UE3, Ordinary differential equations (7 ECTS)

General existence and uniqueness theorems (Grönwall's Lemma, Picard-Lindelöf theorem, global existence, dependence on initial data). Linear differential systems (resolution of autonomous systems, systems with constant coefficients). Nonlinear autonomous systems (general properties of flows and orbits, phase portraits in two dimensions). Stability theorems (notions of stability, asymptotic stability, Lyapunov theorem).

UE4, Quantum physics (7 ECTS)

Introduction: Observables in classical mechanics, finite dimensional model of quantum mechanics. Basic principles of quantum mechanics: States and observables in quantum mechanics, quantum entanglement, Heisenberg uncertainty principle, coordinate and momentum representations. Quantum dynamics: Schrödinger and Heisenberg pictures, Schrödinger equation, classical limit. Quantum mechanics in one dimension: Harmonic oscillator, creation and annihilation operators, scattering problem in one dimension. Quantum mechanics in 3D: Free particle, rotation group and angular momentum, hydrogen atom, spin. Multi-particle quantum systems: Introduction.

Courses of Semester 2 (January - May)

UE6, Groups and representations (6 ECTS)

Notion of a group representation. Development of the structure theory for complex representations of finite groups: Theorems of Maschke and Schur. Tensor products and duality. Character theory. Induced representations. Some outlook beyond finite groups.

UE7, Mathematical methods of classical mechanics (6 ECTS)

Lagrangian and Hamiltonian formalisms. Hamiltonian systems on symplectic manifolds. Variational principle and Hamilton-Jacobi equations. Poisson manifolds. Symmetries and momentum map.

UE8, Numerical methods for physics (6 ECTS)

Interpolation and/or linear systems. Numerical integration (classical rules, Gaussian quadrature rules). Fourier approximation. Numerical methods for solving ODE and PDE.

UE9, Partial differential equations (6 ECTS)

Distributions on $\mathbf{R}^n$: definition, convergence, distributions with compact support and tempered distributions, convolution, Fourier transform. Initial value problems: classical solutions, Fourier method, applications to the heat, wave and Schrödinger equations. Initial boundary value problems: heat operator on a bounded interval, variational formulation of the heat equation.

UE10, Dissertation (6 ECTS)

Students are required to choose a supervisor and a topic during the month of October and to work on the project under the guidance of the supervisor during the whole academic year.